# DEVELOPING MATHEMATICAL MODEL TO DESCRIBE THE RELATIONSHIP BETWEEN SOME PROPERTIES AND FREQUENCY OF VIBRATION

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**ABSTRACT**

**This study summarizes the research work on the effect of mechanical mold vibration during solidification, on some mechanical properties, like tensile strength, elongation, energy stored and hardness. Cylindrical castings were produced from Al-Si alloy in die molds and were subjected to different level of vibration . The vibrated and non vibrated samples were then subjected to tests to determine the above mentioned properties. Based on the values obtained a mathematical model was developed describing the relationship between the properties and frequency of vibration. The result indicates the presence of vibration has a major contribution in affecting the responses measured.**

**Key Words: Mathematical Model, Relationship, Frequency, Properties, Vibration**

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## INTRODUCTION

Aluminium-Silicon alloys are one of the most commonly used foundry alloys because they offer many advantages such as good thermal conductivity,excellent castability,high strength to weight ratio,wear and corrosion resistance,etc. Therefore they are well suited to automotive cylinder heads,engine blocks,aircraft components etc. The mechanical properties of Auminium-Silicon alloys are related to the grain size and shape. Imposition of vibration on liquid metal during solidification has shown several advantages like grain refinement, increased density, degassing,shrinkage and improvement of mechanical properties.Grain structure of casting changes from columnar dendritic to equiaxed dendrites or globular. It has been observed that in order to get pronounced grain refinement, the solidifying melt should be kept under the influence of vibration energy for reasonably long time ranging from 1-5 minutes [1]. This can be done by choosing alloys with long freezing range or preheating the mold.

Different methods have been used to apply vibrations during solidification. Electromagnetic vibration is one of the non contact methods used to induce vibration in the solidifying metal [2]. Several other researchers have investigated the effect of vibration on the microstructure of castings [4-6]. The effects include grain refinement, fragmentation of the dendrite structure and degassing resulting in reduced porosity. Pandel et al [7] have reported reduction in average size of silicon needles with increase in amplitude of vibration from 1- 3 mm, for hypoeutectic and hyper eutectoid Al-Si alloys. Burbure et al [8] have reported grain refinement in aluminum casting solidified under the influence of low frequency vibration of 50 Hz. The refinement was more pronounced with increase in casting size and at lower initial mold temperature. Abu-Dheir et al [9] have reported increase in percent elongation in the castings subjected to vibration of 100 Hz and varying amplitude of

18-199 micron. Also increase in amplitude has been reported to reduced interlamellar spacing between silicon needles. In another study, Kadir Kocatepe [10] reported that the amount and size of pores were increased in Al-Si alloys with increasing frequencies. .

Thus, it is clear that vibration promotes changes in microstructure and consequently in mechanical properties.

The present work has been carried out to study the effect of vibration during solidification on some mechanical properties and to also determine a mathematical model to predict same. This work was limited to a range of frequencies of between 1hz-10hz.



METHODOLOGY

The nth degree polynomial equation is generally expressed as;

$$f(x) = C_0 + C_1 x + C_2 X^2 + \ \dots\dots + C_{N-1} X^{N-1} + C_N X^N \qquad (1)$$

Where $C_N$ are polynomial coefficients.

The values of the coefficients could be obtained by substituting the boundary conditions as obtained from the experiment.

**EXPERIMENTAL PARAMETERS (as boundary conditions)**

| Frequency of Vibration (Hz) | Mechanical Properties | | | |
|---|---|---|---|---|
| | Ultimate Tensile strength, N/sq. mm | Energy Stored | % Elongation | Hardness |
| 1 | 31.91 | 0.04074 | 0.0884 | 8.988732 |
| 3 | 157.12 | 1.1907 | 0.023 | 44.25915 |
| 5 | 193.32 | 1.89998 | 0.024 | 54.45633 |

**DEVELOPING TENSILE STRENGHT EQUATION**

Let the tensile strength be $\sigma_T$ which is y on the curve and the frequency of vibration be $\lambda$ which is x on the curve. Equation (1) could be adopted for a second degree polynomial as expressed in equation (2) to satisfy the three boundary conditions;

$$\sigma_T = C_0 + C_1\lambda + C_2\lambda^2 \qquad (2)$$

Substituting the boundary conditions,

At $\lambda = 1,\ \sigma_T = 31.91$

$$C_0 + C_1 + C_2 = 31.91 \qquad (3A)$$

At $\lambda = 3,\ \sigma_T = 157.12$

$$C_0 + 3C_1 + 9C_2 = 157.12 \qquad (3B)$$

At $\lambda = 5,\ \sigma_T = 193.32$

$$C_0 + 5C_1 + 25C_2 = 193.32 \qquad (3C)$$

Equations 3A – 3C can be presented in matrix form as follows:

$$\begin{pmatrix} 1 & 1 & 1 & | & 31.91 \\ 1 & 3 & 9 & | & 157.12 \\ 1 & 5 & 25 & | & 193.32 \end{pmatrix}$$

Keeping row 1 and row 2 and subtracting row 1 from row 3 to develop new row 3;

$$\begin{pmatrix} 1 & 1 & 1 & | & 31.91 \\ 1 & 3 & 9 & | & 157.12 \\ 0 & 4 & 24 & | & 161.41 \end{pmatrix}$$

Keeping row 1 and row 3 and subtracting row 1 from row 2 to develop new row 2;

$$\begin{pmatrix} 1 & 1 & 1 & | & 31.91 \\ 0 & 2 & 8 & | & 125.21 \\ 0 & 4 & 24 & | & 161.41 \end{pmatrix}$$

Keeping row 1 and row 2 and subtracting 2x(row 2) from row 3 to develop new row 3;

$$\begin{pmatrix} 1 & 1 & 1 & | & 31.91 \\ 0 & 2 & 8 & | & 125.21 \\ 0 & 0 & 8 & | & -89.01 \end{pmatrix}$$

Considering row 3 this implies that;

$8\ C_2 = -89.01$ therefore,

$$C_2 = \frac{-89.01}{8} = -11.126$$

Considering row 2 this implies that;

$2\ C_1 + 8\ C_2 = 125.21$

Substituting value of $C_2$ and calculating for $C_1$

$C_1 = 107.109$

Considering row 1

$C_0 + C_1 + C_2 = 31.91$

Substituting values of $C_2$ and $C_1$ and calculating $C_0$

$C_0 = -64.073$

Thus

$C_2 = -11.126$

$C_1 = 107.11$

$C_0 = -64.074$

Hence the polynomial equation for the Tensile strength curve is expressed by substituting values of the coefficients into equation (2).

$$\sigma_T = -11.126\lambda^2 + 107.11\lambda - 64.074$$

**Note; in the curve $\boldsymbol{\lambda = x\ and\ \sigma_T = y}$**

**DEVELOPING ENERGY STORED EQUATION**

Let the energy stored be E which is y on the curve and the frequency of vibration be $\lambda$ which is x on the curve. Equation (1) could be adopted for a second degree polynomial as expressed in equation (2) to satisfy the three boundary conditions;

$$E = C_0 + C_1\lambda + C_2\lambda^2 \tag{4}$$

Substituting the boundary conditions,

At $\lambda = 1,\ E = 0.04074$

$$C_0 + C_1 + C_2 = 0.04074 \tag{4A}$$

At $\lambda = 3,\ E = 1.1907$

$$C_0 + 3C_1 + 9C_2 = 1.1907 \tag{4B}$$

At $\lambda = 5,\ E = 1.89998$

$$C_0 + 5C_1 + 25C_2 = 1.89998 \tag{4C}$$

Equations 3A – 3C can be presented in matrix form as follows:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0.04074 \\ 1 & 3 & 9 & 1.1907 \\ 1 & 5 & 25 & 1.89998 \end{array}\right)$$

Keeping row 1 and row 2 and subtracting row 1 from row 3 to develop new row 3;

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0.04074 \\ 1 & 3 & 9 & 1.1907 \\ 0 & 4 & 24 & 1.85924 \end{array}\right)$$

Keeping row 1 and row 3 and subtracting row 1 from row 2 to develop new row 2;

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0.04074 \\ 0 & 2 & 8 & 1.14996 \\ 0 & 4 & 24 & 1.85924 \end{array}\right)$$

Keeping row 1 and row 2 and subtracting 2x(row 2) from row 3 to develop new row 3;

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0.04074 \\ 0 & 2 & 8 & 1.14996 \\ 0 & 0 & 8 & -0.44068 \end{array}\right)$$

Considering row 3 this implies that;

$8\ C_2$ =-0.44068 therefore,

$$C_2 = \frac{-0.44068}{8} = -0.055085$$

Considering row 2 this implies that;

$2\ C_1 + 8\ C_2$=1.14996

Substituting value of $C_2$ and calculating for $C_1$

$C_1$= 0.79498

Considering row 1

$C_0 + C_1 + C_2$ =0.04074

Substituting values of $C_2$ and $C_1$ and calculating $C_0$

$C_0$ = -0.69924

Thus

$C_2 = -0.055$

$C_1 = 0.79498$

$C_0 = -0.69924$

Hence the polynomial equation for the Tensile strength curve is expressed by substituting values of the coefficients into equation (2).

$$E = 0.69924\lambda^2 + 0.79498\lambda - 0.69924$$

**Note; in the curve $\boldsymbol{\lambda = x\ an}$**



**DEVELOPING % elongation EQUATION**

Let the %elongation be e which is y on the curve and the frequency of vibration be $\lambda$ which is x on the curve. Equation (1) could be adopted for a second degree polynomial as expressed in equation (2) to satisfy the three boundary conditions;

$$e = C_0 + C_1\lambda + C_2\lambda^2 \quad (5)$$

Substituting the boundary conditions,

At $\lambda = 1,\ e = 0.0884$

$$C_0 + C_1 + C_2 = 0.0884 \quad (5A)$$

At $\lambda = 3,\ e = 0.023$

$$C_0 + 3C_1 + 9C_2 = 0.023 \quad (5B)$$

At $\lambda = 5,\ e = 0.024$

$$C_0 + 5C_1 + 25C_2 = 0.024 \quad (5C)$$

Equations 3A – 3C can be presented in matrix form as follows:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0.0884 \\ 1 & 3 & 9 & 0.023 \\ 1 & 5 & 25 & 0.024 \end{array}\right)$$

Keeping row 1 and row 2 and subtracting row 1 from row 3 to develop new row 3;

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0.0884 \\ 1 & 3 & 9 & 0.023 \\ 0 & 4 & 24 & -0.0644 \end{array}\right)$$

Keeping row 1 and row 3 and subtracting row 1 from row 2 to develop new row 2;

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0.0884 \\ 0 & 2 & 8 & -0.0614 \\ 0 & 4 & 24 & -0.0644 \end{array}\right)$$

Keeping row 1 and row 2 and subtracting 2x(row 2) from row 3 to develop new row 3;

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0.0884 \\ 0 & 2 & 8 & -0.0654 \\ 0 & 0 & 8 & 0.0584 \end{array}\right)$$

Considering row 3 this implies that;

$8\ C_2 = 0.0584$ therefore,

$$C_2 = \frac{0.0584}{8} = 0.0073$$

Considering row 2 this implies that;

$2\ C_1 + 8\ C_2 = -0.0654$

Substituting value of $C_2$ and calculating for $C_1$

$C_1 = -0.0619$

Considering row 1

$C_0 + C_1 + C_2 = 0.0884$

Substituting values of $C_2$ and $C_1$ and calculating $C_0$

$C_0 = 0.143$

Thus

$$C_2 = 0.0073$$

$$C_1 = -0.0619$$

$$C_0 = 0.143$$

Hence the polynomial equation for the Tensile strength curve is expressed by substituting values of the coefficients into equation (2).

$$e = 0.0073\lambda^2 - 0.0619\lambda + 0.143$$

**Note; in the curve $\lambda = x$ and $e =$**

## DEVELOPING HARDNESSEQUATION

Let the hardness be H which is y on the curve and the frequency of vibration be $\lambda$ which is x on the curve. Equation (1) could be adopted for a second degree polynomial as expressed in equation (2) to satisfy the three boundary conditions;

$$H = C_0 + C_1\lambda + C_2\lambda^2 \quad (6)$$

Substituting the boundary conditions,

At $\lambda = 1,\ H = 8.989$

$$C_0 + C_1 + C_2 = 8.989 \qquad (6A)$$

At $\lambda = 3,\ H = 44.259$

$$C_0 + 3C_1 + 9C_2 = 44.259 \qquad (6B)$$

At $\lambda = 5, H = 54.456$

$$C_0 + 5C_1 + 25C_2 = 54.456 \qquad (6C)$$

Equations 3A – 3C can be presented in matrix form as follows:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 8.989 \\ 1 & 3 & 9 & 44.259 \\ 1 & 5 & 25 & 54.456 \end{array}\right)$$

Keeping row 1 and row 2 and subtracting row 1 from row 3 to develop new row 3;

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 8.989 \\ 1 & 3 & 9 & 44.259 \\ 0 & 4 & 24 & 45.467 \end{array}\right)$$

Keeping row 1 and row 3 and subtracting row 1 from row 2 to develop new row 2;

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 8.989 \\ 0 & 2 & 8 & 35.27 \\ 0 & 4 & 24 & 45.467 \end{array}\right)$$

Keeping row 1 and row 2 and subtracting 2x(row 2) from row 3 to develop new row 3;

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 8.989 \\ 0 & 2 & 8 & 35.27 \\ 0 & 0 & 8 & -25.073 \end{array}\right)$$

Considering row 3 this implies that;

$8\ C_2 = -25.073$ therefore,

$$C_2 = \frac{-25.073}{8} = -3.134125$$

Considering row 2 this implies that;

$2\ C_1 + 8\ C_2 = 35.27$

Substituting value of $C_2$ and calculating for $C_1$

$C_1 = 30.1715$

Considering row 1

$C_0 + C_1 + C_2 = 8.989$

Substituting values of $C_2$ and $C_1$ and calculating $C_0$

$C_0 = -18.0484$

Thus

$$C_2 = -3.134125$$

$$C_1 = 30.1715$$

$$C_0 = -18.0484$$

Hence the polynomial equation for the Tensile strength curve is expressed by substituting values of the coefficients into equation (2).

$$H = -3.134125\lambda^2 + 30.1715\lambda - 18.0484$$

**Note; in the curve $\boldsymbol{\lambda = x \text{ and } H = y}$**

**RELATIONSHIP BETWEEN PROPERTIES**

**In order to determine the relationship between properties and frequency, a mathematical function (curve fitting) that has the best fit to the series of data obtained anterior from experiment for determined range was constructed subject to constraints.**

## Input data:

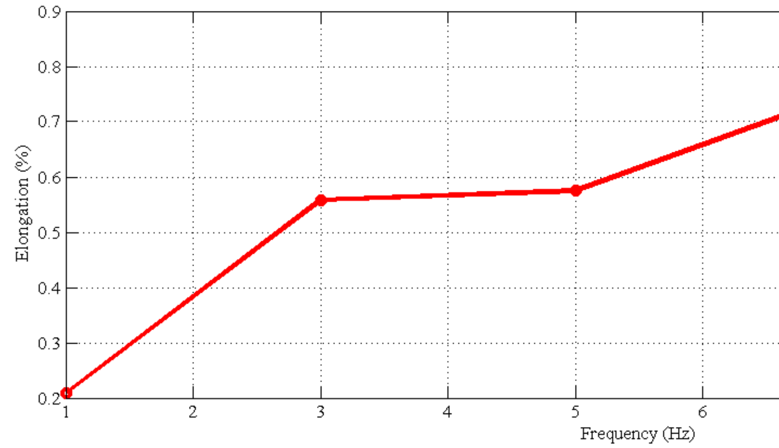
**Frequency of vibration:** f = [1; 3; 5; 7; 10] (Hz).

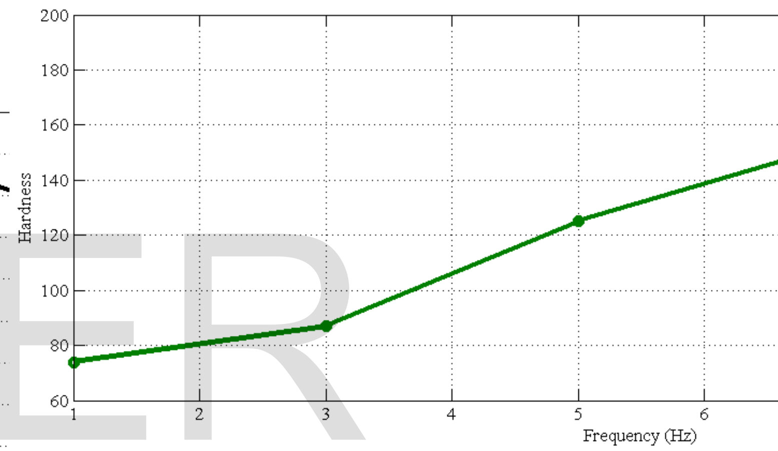
**Tensile Strength:** σ = [31.91; 157.12; 193.32; 155.19; 147.775].

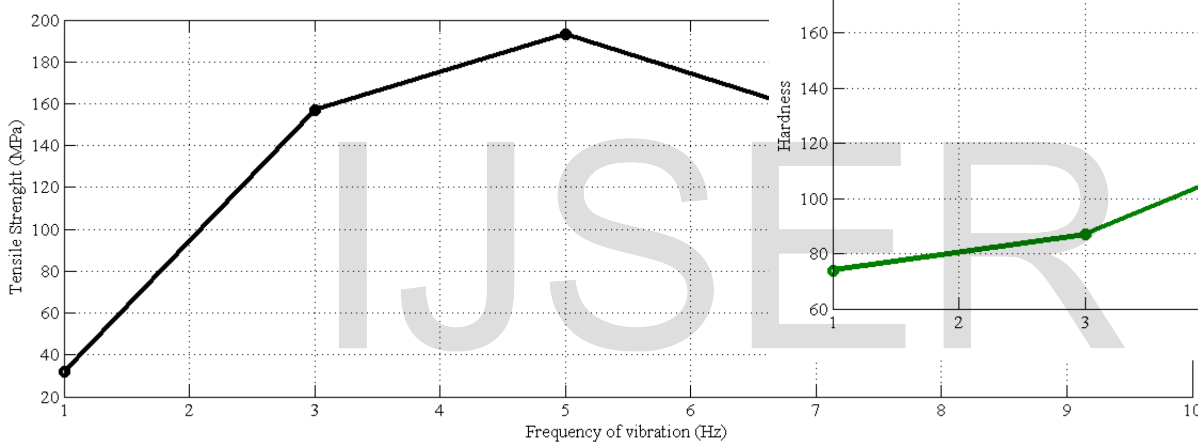
**Energy stored:** E = [0.04074; 1.1907; 1.89998; 19.543; 13.50875].

**Elongation:** e = [0.20819 0.55825 0.57506 0.7416 0.633312].

**Hardness:** H = [74, 87, 125, 152, 185].

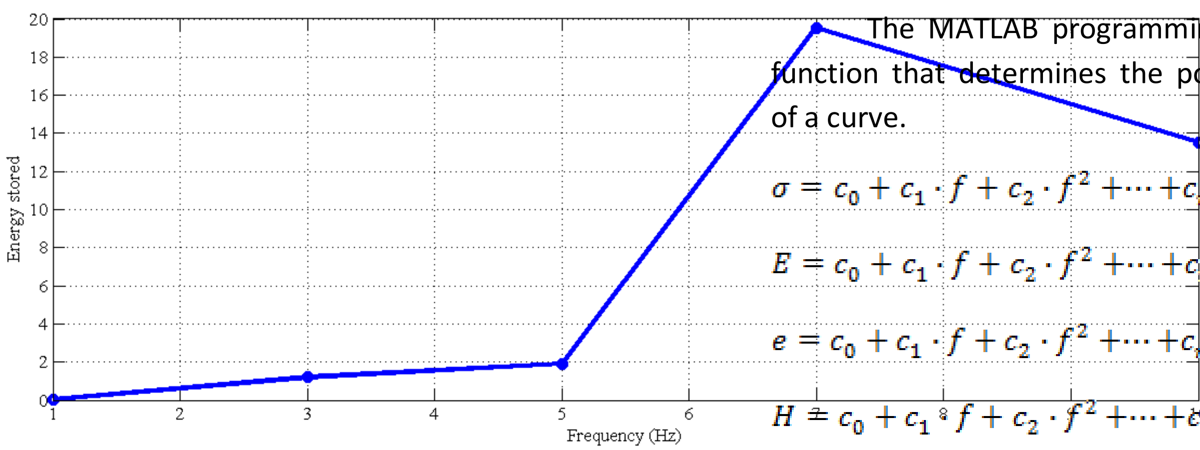





## Programmatic Curve Fitting




The MATLAB programming software has a function that determines the polynomial function of a curve.

$$\sigma = c_0 + c_1 \cdot f + c_2 \cdot f^2 + \cdots + c_n \cdot f^n$$

$$E = c_0 + c_1 \cdot f + c_2 \cdot f^2 + \cdots + c_n \cdot f^n$$

$$e = c_0 + c_1 \cdot f + c_2 \cdot f^2 + \cdots + c_n \cdot f^n$$

$$H = c_0 + c_1 \cdot f + c_2 \cdot f^2 + \cdots + c_n \cdot f^n$$

*Poly f(x,y,n),* function - finds the coefficients of a polynomial p(x) of degree n that fits the y data by minimizing the sum of the squares of the deviations of the data from the model (least-squares fit).

*Poly Val(p,x)* - returns the value of a polynomial of degree n that was determined by polyfit, evaluated at x.

After running the program the following equations were determined:

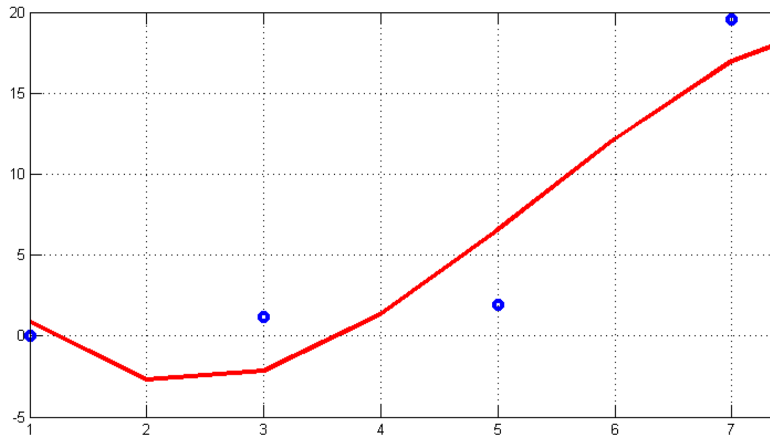
**Tensile Strength**

c = [1.1818 -24.0970 146.8439 -93.3420]

$$\sigma = 1.1818 \cdot f^3 - 24.097 \cdot f^2 - 93.342$$

c = [-0.2084 3.3410 -12.1723 9.9422]

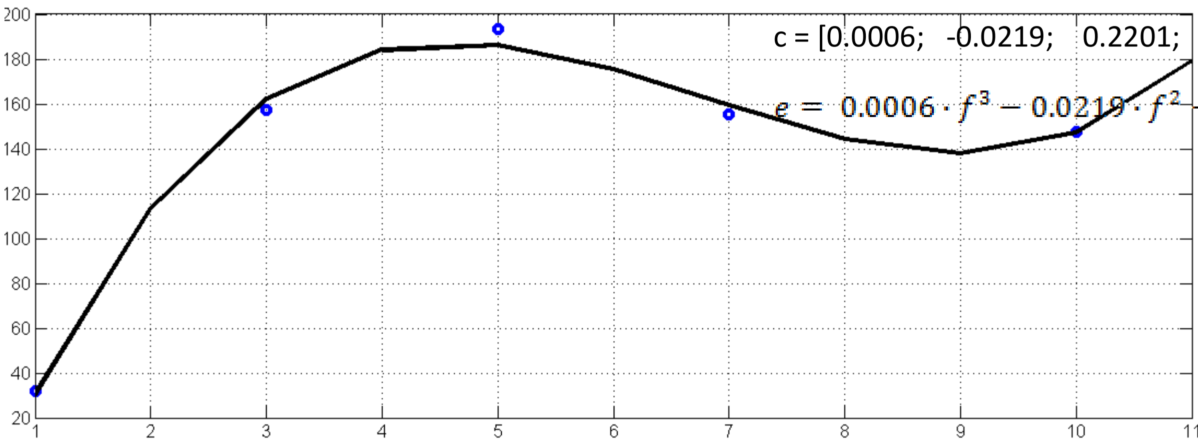
$$E = -0.2084 \cdot f^3 + 3.341 \cdot f^2 - 12.1723 \cdot f + 9.9422$$



**Elongation**

c = [0.0006; -0.0219; 0.2201; 0.0237]

$$e = 0.0006 \cdot f^3 - 0.0219 \cdot f^2 + 0.2201 \cdot f + 0.0237$$



**Energy stored**

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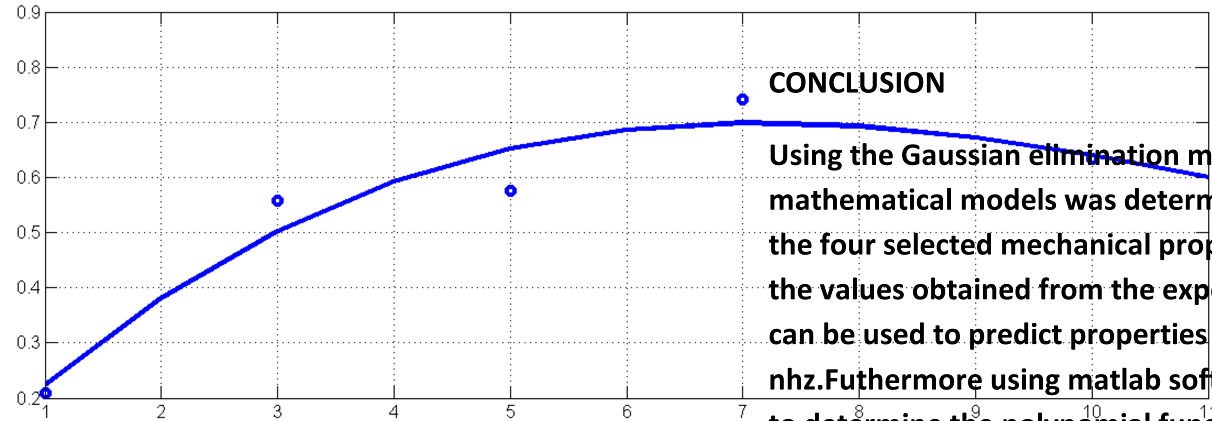
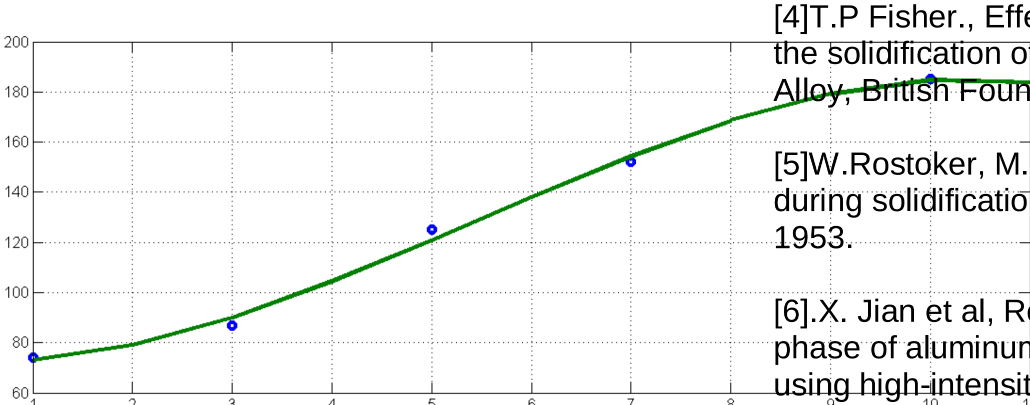



**Hardness:**

c = [-0.2375; 3.8967; -4.1188; 73.6854]

$$H = -0.2375 \cdot f^3 + 3.8967 \cdot f^2 - 4.1188 \cdot f + 73.6854$$




**CONCLUSION**

**Using the Gaussian elimination method mathematical models was determined for each of the four selected mechanical properties based on the values obtained from the experiment and this can be used to predict properties between 1hz-nhz.Futhermore using matlab software was used to determine the polynomial function for the prediction of the properties in the range of 1hz - 10HZ**

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